A Consideration on Visualization of Temporal Signals by Feedback SOM

Hiroshi Wakuya, Shohei Matsuoka, Hideaki Itoh
Graduate School of Science and Engineering, Saga University
Honjo-machi, Saga 840-8502, Japan
Email: wakuya@cc.saga-u.ac.jp, {matsuoka, hideaki}@ace.ec.saga-u.ac.jp

Abstract—An Elman-type feedback SOM (EFSOM) is a modified version of the standard SOM for temporal signal processing. It was applied to various kinds of tasks so far, and it showed good performance on temporal signal processing. By the way, one of the major features inspired by the standard SOM is to visualize given data structure into the two-dimensional plane preserving its topology. Then, from the visualization point of view, a working mechanism of the feedback SOM (FSOM), which is an essential part of the EFSOM, is investigated in this paper. As a result of computer simulations, a trajectory of the winner neurons is bifurcated, not overlapped between any two segments, with the help of feedback pathways. This fact implies that the FSOM can deal with the past history, i.e., the context information, appropriately. Furthermore, when continuous pattern is used for training, it is found that signal processing developed in the SOM is quite similar to that for an A/D converter based on the discretized reference vectors.

I. INTRODUCTION

Originally, a self-organizing map (SOM) is a neural network model based on the biological visual systems [1]. Since there are no dynamical elements in it, the standard SOM cannot deal with any time-variant information intrinsically. A simple way to overcome this disadvantage is to convert a given temporal pattern to a spatial pattern with unit-delay elements as an input signal. Another way is to replace static neurons with dynamic neurons, e.g., leaky integrators [2], [3]. Also, providing feedback pathways around the competitive layer is worth considering [4].

Following to the above-mentioned trend in temporal signal processing by SOM architecture, an Elman-type feedback SOM (EFSOM) was proposed [5]. It was applied to various kinds of tasks, including a Braille recognition task [5], an on-line character recognition task [5], an on-line character recognition task [6], [7] and so on, and it showed good performance on temporal signal processing for both temporal elasticity and spatial displacement.

By the way, one of the major features inspired by the standard SOM is to visualize given data structure into the two-dimensional plane preserving its topology. Even though the EFSOM has a such distinct feature, it has been concentrated on its performance mainly. It is true that an analysis based on the neuro-bar model [8] was carried out, but its approach was not sufficient to make clear the mechanism. Then, from the visualization point of view, a working mechanism of the feedback SOM (FSOM), which is an essential part of the EFSOM, is investigated in this paper.

In the following part, an overview of the FSOM for temporal signal processing is explained in Section II. Secondly, some applications we tried in the preceding studies is presented in Section III. Thirdly, some computer simulations are carried out in Sections IV and V. Fourthly, discussion from the viewpoints of the further considerations is made in Section VI. Finally, conclusions from this study are summarized in Section VII.

II. FEEDBACK SOM

Fig.1 shows an architecture of the FSOM. It is an essential part of the EFSOM, and it corresponds to a modified version of the original FSOM proposed by Horio and Yamakawa [4]. Basically, it is a standard SOM which has feedback pathways around the competitive layer. It consists of three layers, i.e., an input layer $x(t)$, a competitive layer $y(t)$, and a state layer $h(t)$. Each layer has $L$, $M$, $M$ neurons, respectively. Then, their signal transmission equations at time $t$ are defined as follows:

$$d_j(t) = \sum_{i=1}^{L} (x_i(t) - w_{ji})^2 + \sum_{i=1}^{M} (\beta h_i(t) - w_{ji,L+i})^2$$

$$d_j(t) \equiv \sum_{i=1}^{L+M} (I_i(t) - w_{ji})^2 , \quad (1)$$

$$I(t) = \{x_1(t), \cdots, x_L(t); \beta h_1(t), \cdots, \beta h_M(t)\} , \quad (2)$$

$$j^* = \arg \min_{1 \leq j \leq M} d_j(t) , \quad (3)$$
\[ y_j(t) = \begin{cases} 
1, & j = j^* \\
0, & j \neq j^* 
\end{cases} \]

where \( I(t) \) is a net input layer composed of both \( x(t) \) and \( h(t) \), \( j^* \) is an index number of the winner neuron in the competitive layer, \( \beta \) is a weighting constant for referring the past history, and \( \gamma \) is a decay constant of the past history \(^{1}\).

In the training phase, all reference vectors \( w_j \) between the net input layer \( I(t) \) and the \( j \)-th neuron in the competitive layer \( y(t) \) are developed as,

\[
w_{ji}(n) = w_{ji}(n-1) + \eta A(j,j^*) \{ I(t) - w_{ji}(n-1) \}
\]

where \( \eta \) is a constant for training speed, \( A(j,j^*) \) is a neighborhood function whose center is the coordinates of the winner neuron \( j^* \). Both of them are provided smaller and smaller as the training epoch number \( n \) increases. In short, training procedures for the FSOM are as the same as that for the conventional standard SOM, i.e., the reference vector \( w_{ji} \) for the winner neuron \( j^* \) is developed to approach the applied input vector \( x \) more and more.

### III. Overview of Preceding Studies

As mentioned above, the EFSOM has been applied to various kinds of tasks. First of all, a Braille recognition task was tried \([5]\). Even though only four kinds of city names were adopted, good performance on temporal signal processing for both temporal elasticity and spatial displacement was confirmed. Next, an on-line character recognition task was tried \([6]\), \([7]\), and the number of training samples was increased. In general, above-mentioned modification makes training more difficult. Its major reasons are as follows:

1) Only penpoint information is given while we are writing a character, so it is required to preserve its past history.

2) The initial state of the EFSOM is the same as a start point for all training samples, so it is required to discriminate them each other based on different inner representations.

It is noticeable here that these aspects are distinct nature for temporal signal processing. In the series of previous studies, an idea called multi-winner neuron style \([6]\), \([7]\) was introduced, and good performance on temporal signal processing for both temporal elasticity and spatial displacement was also confirmed.

As described above, it has been concentrated on its performance mainly. But one of the major features inspired by the standard SOM is to visualize given data structure into the two-dimensional plane. Then, from the visualization point of view, a working mechanism of the FSOM is investigated.

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\(^{1}\) In other words, \( \gamma \) corresponds to a time constant \( \tau \) for first order decay element: \( \gamma = \exp(-1/\tau) \).

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A temporal pattern used for training.

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IV. Computer Simulations \([1]\)

— HOW TO TRAIN THE TEMPORAL PATTERNS —

A. Methods

In order to investigate a working mechanism of the FSOM, a simple temporal pattern is prepared for training. As can be seen in Fig.2(a), it is divided into two main parts. The former one is a constant part to converge on a steady state, which corresponds to the start point mentioned above. It is a null input part for 100 steps long, and it plays an important role in removing any clues for identifying the objective characters in advance. In contrast, the latter one is a cyclic part in the range of \([-1.0, 1.0]\). It is a sinusoidal input part, whose amplitude \( A \) is set to 1.0, for 32 steps long. According to Fig.2(b), the
cyclic part consists of four 8-step-long segments. The first two segments are positive, while the last two segments are negative. From the viewpoint of alignment of applying values, each two segments are symmetry. Then, they are referred to as forward segment and backward segment hereafter, respectively.

By the way, digital patterns whose components are discrete (0/1) have been adopted in the preceding studies mainly. Then, an effect of analog patterns whose components are continuous are adopted to identify the behaviors against the intermediate values.

In this study, a single temporal pattern is used for training, so the FSOM is designed as \( L = 1 \) and \( M = 400 \) (=20x20). Fig.3 is an actual structure of the FSOM used in this study, i.e., a substituted version of the actual parameters in Fig.1 for easy to understand. The other parameters are determined as \( \beta = 0.0014 \) and \( \gamma = 0.2 \) following the result of some preliminary experiments. In order to compare their performance, the same structure of the standard SOM without any feedback pathways is also prepared.

**B. Results**

1) **Trajectory of the Winner Neurons:** After the training phase for the FSOM is completed, a trajectory of the winner neurons in the 20x20-wide competitive layer is observed. Fig.4 shows an example of trajectories when the temporal pattern used for training is applied. Each filled circle shows the location of the emerging winner neuron. At first glance, it is found that the trajectory is bifurcated, not overlapped between the forward and backward segments. In other words, it is said that the FSOM can train the temporal pattern successfully, which we have intended in advance, with the help of feedback pathways. Although not shown here for brevity, a trajectory of the winner neurons for the standard SOM is not bifurcated, because it cannot discriminate between the forward and backward segments intrinsically.

2) **Inter-Neuronal Weights Developed through Training:** As a next step, behavior of the FSOM is considered. In general, responses of all neurons are determined uniquely based on both the applied input signal and the inter-neuronal weights. It is obvious that the applied input signal is the same as that used for training. Therefore, acquired inter-neuronal weights are observed in two ways. One is the weights \( w_{j1} \) from the single input neuron \( x_1 \) to all neurons \( y_j(j = 1, 2, \cdots, 400) \) in the competitive layer. As mentioned in Section II, the reference vector of the winner neuron is developed to approach the applied input vector, therefore it must be easy to identify where the winner neuron will appear. The other is the weights \( w_{j,i+1} \) from all neurons \( h_i(i = 1, 2, \cdots, 400) \) in the state layer to a single neuron \( y_j \) in the competitive layer. In this case, there are several versions depending on the neuron \( y_j \) which is selected in the competitive layer. If arbitrary two weights \( w_{j1} \) and \( w_{j1'} \) are the same strength under the condition \( j \neq j' \), difference between the weights \( w_{j1} \) and \( w_{j1'} \) for \( i = 1, 2, \cdots, 400 \) must be significant to discriminate them each other from the viewpoint of preserving past history, i.e., context information. For reference, above-mentioned two kinds of weights are summarized in Fig.5(a)(b) with the bold lines, respectively.

Fig.6 is the former case, and it is confirmed that all weights are aligned smoothly from -1.0 to 1.0 based on their strength. Its tendency is changing along with the diagonal direction, because spatial resolution of the competitive layer will be magnified.

Fig.7 is the latter case, and some examples shown in Fig.6(a) are selected because they belong to the same category. It is clear that distributions of weights are different each other, so this fact suggests that the context information the FSOM possesses here is preserved appropriately.

3) **Other Extra Trials Carried Out in This Study:** According to Fig.6, the applied continuous input signal is compared with all discretized reference vectors, and then a neuron
whose reference vector is the closest is selected to become a winner neuron. A series of these operations is quite similar to that for an analog-to-digital (A/D) converter, so an inverse conversion is tried based on the weights from the winner neurons in the competitive layer to the single input neuron. As can be seen in Fig. 8, it is obvious that the applied input signal is reconstructed appropriately. In this case, discretization is developed adaptively through training, so that each step width must be optimized just fit for the temporal pattern used for training.

V. Computer Simulations [II] — Analysis of Acquired Weights —

A. Methods

In order to investigate the acquired weights developed through training in Section IV, another temporal patterns with different amplitude $A$ are prepared. Its range is varied from 0.2 to 2.0 at 0.2 intervals. It is noticeable here that all of them except $A = 1.0$ are completely new to the trained FSOM. Then, in order to estimate its working mechanism, responses of the FSOM are observed.

B. Results

Fig. 9 shows the trajectories of the winner neurons when various kinds of temporal patterns are applied to the trained FSOM. It is obvious that the start point is the same as that shown in Fig. 4.

In the cases for $A < 1.0$, some neurons are assigned as the common winner neuron for succeeding several input signals, and there are no winners at the top/bottom of the weight landscape shown in Fig. 4. It is a “quantization effect” based on the A/D conversion as mentioned above, because each step width is determined through training automatically to compare with the applied signal. Then, the winner neuron tries to climb up the hill, but it turns to climb down before reaching the top, and vice versa.

In the cases for $A > 1.0$, in contrast, the winner neuron
Fig. 9 Results of computer simulations [IV] — Trajectory of the winner neurons for untraining patterns.

VI. Discussion

In general, numerous studies in the field of SOM have been undertaken by a lot of researchers so far. But most of them seem to be adopted discrete patterns. For example, some of them are binary patterns (0/1), and the others are bipolar patterns (-1/1). On the contrary, in order to investigate the working mechanism of the FSOM, a continuous pattern in the range of [-1.0, 1.0] is adopted in this study. Even though its original motivation is quite simple, and it is just to identify the behaviors against the intermediate values, various kinds of fruitful results have been found.

At first, trajectories of the winner neurons in the FSOM are bifurcated between the forward and the backward segments. Its strategy is easy to understand for preserving the context information, and it also reminds us a hysteresis loop often used in the field of electromagnetic theory. In other words, the standard SOM cannot preserve the context information intrinsically, so trajectories of the winner neurons are not bifurcated.

Secondly, signal processing developed in the SOM through training is quite similar to that for the A/D converter. If a continuous input signal is applied to the trained SOM, it is compared with all discretized reference vectors in it. And then, a neuron whose reference vector is the closest is selected to become a winner neuron. As mentioned above, a series of these operations is quite similar to that for the A/D converter. Once discretization of reference vectors is made successfully, each applied signal might be coded as a location where the corresponding winner neuron will be appear. But one of its disadvantages is meaningless against any untraining patterns. As can be seen in Fig.9, this is why discretization is not always fit for any applied patterns.

VII. Conclusions

In this paper, a working mechanism of the feedback SOM (FSOM) is investigated from the visualization point of view. As a result of computer simulations, a trajectory of the winner neurons is bifurcated, not overlapped between any two segments, with the help of feedback pathways. It is found that the FSOM can deal with the past history, i.e., the context information, appropriately. Furthermore, when continuous pattern is used for training, it is also found that signal processing developed in the SOM is quite similar to that for an A/D converter based on the discretized reference vectors.
References


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