Adaptive Noise Cancelling Based on Independent Component Analysis

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Indexing terms: Adaptive signal processing, independent component analysis,
adaptive noise cancelling

A method for adaptive noise cancelling based on independent com-ponent analysis (ICA) is presented. Although conventional least-mean-squares (LMS) algorithm removes noise components based on second-order correlation, the proposed algorithm can utilize higher-order statistics. Experimental results show that the proposed algorithm provides considerable performance improvement.
**Introduction:** Adaptive noise cancelling is an approach to reduce noise based on reference noise signals \([1]\). In conventional adaptive noise cancelling systems, the primary input signal is a combined signal and noise \(s + n_0\) and the reference signal is a noise signal \(n_1\) through another channel from the same noise source. The goal is to get a system output \(u\) which is the best least squares estimate of the signal \(s\). The most popular algorithm for noise cancellation is LMS algorithm \([1]\), which adapts as

\[
\Delta w(k) \propto u(t)n_1(t - k),
\]

where the output \(u\) is

\[
u(t) = s(t) + n_0(t) - \sum_{k=1}^{K} w(k)n_1(t - k).
\]

It decorrelates system output signal from the reference noise signal and removes noise components of the primary input signal based on second-order statistics only. However, there may exist many other components in the primary input signal which depend on the noise reference signal in higher-order statistics. In this letter, we present a method that can remove noise components based on statistical independence, which incorporates statistics of all orders.

**Learning rule:** ICA was proposed to recover independent sources from given sensor signals in which the sources have been mixed through unknown channels \([2][3]\). Learning rules of adaptive filter coefficients in the noise cancellation system can be derived by maximizing entropy. To derive the learning rules conveniently, we set dummy output \(v = n_1\). It doesn’t make any difference during derivation using...
entropy maximization because noise $n_1$ is independent of signal $s$. In this system, the Jacobian can be expressed as

$$J = \frac{\partial y_1}{\partial x} \frac{\partial y_2}{\partial n_1} - \frac{\partial y_1}{\partial n_1} \frac{\partial y_2}{\partial x} = \frac{\partial y_1}{\partial u} \frac{\partial y_2}{\partial v}$$

(3)

where $x = s + n_0$, and $y_1$ and $y_2$ are outputs of nonlinear functions approximating the cdfs of the signal and the noise, respectively. By maximizing $\log|J|$, the learning rule of each coefficients can be obtained as follows:

$$\Delta w(k) \propto \frac{\partial}{\partial w(k)} \log|J| = \varphi(u(t))n_1(t - k),$$

(4)

where the score function $\varphi(u)$ is

$$\varphi(u) = -\left(\frac{\partial y_1}{\partial u}\right)^{-1} \frac{\partial^2 y_1}{\partial u^2}.$$  

(5)

The difference between the LMS algorithm and the ICA-based approach comes from existence of the score function. Introducing nonlinearity to the LMS algorithm has been studied by many researchers to improve the properties and the performances [4][5]. Especially, Douglas and Meng generalized the LMS algorithm by using a nonlinearity acting on the error for system identification problem [4]. They provided methods for optimizing the nonlinearity to minimize the misadjustment for a given convergence rate. Under i.i.d. signal assumption, the same nonlinearity was derived using linearized approximation near convergence, but in general derivation, nonlinearity was different assuming zero-mean white Gaussian (noise) data. However, the ICA-based learning rule can be derived with independence between the signal and the noise sources without any other assumptions.
The LMS algorithm decorrelates output signal from the reference input to remove corrupting noise component which is correlated to the reference input. However, the ICA-based approach makes output signal independent of the reference input. The independence involves statistics of all orders including the second-order statistics, i.e. correlation. When noise signals are obtained from a noise source through different channels, there may exist many components which depend on each other through higher-order statistics. The ICA-based approach can be used for cancelling these noise components and provide better performances than the conventional LMS algorithm.

Experimental Results: We have compared the performances of the ICA-based approach with those of the LMS algorithm in terms of signal-to-noise ratio (SNR), which we define for the output \( u \) in the typical adaptive noise cancelling system as the total power of the components caused by the signal source versus that caused by the noise source,

\[
\text{SNR} = \frac{\langle (s(t))^2 \rangle}{\langle (n_0(t) - \sum_{k=1}^{K} w(k)n_1(t - k))^2 \rangle}.
\]

The transfer functions from the signal source to the primary input and from the noise source to the reference input are simple linear scales, and the proper scale values were chosen to obtain desired initial SNRs. Two different mixing filters were used as the filter \( h_{12} \) from the noise source to the primary input. Assuming that the primary and the reference inputs pick up signals with appropriate powers, we have normalized mixture powers properly (generally to 1). And, all experiments
were conducted with several step sizes, and the best performance is shown.

Table 1 displays the SNRs of output signals for artificially generated i.i.d. sig-
nals. A simple simulation filter was chosen as the filter $h_{12}$ as follows [3]:

$$h_{12}(z) = 0.4z^{-20} - 0.2z^{-28} + 0.1z^{-36}. \quad (7)$$

And, the number of taps of adaptive filter coefficients was 128. Each signal was
composed of 160000 samples. For the ICA-based approach, two different score
functions were used. $\text{sign}(\cdot)$ and $\text{tanh}(\cdot)$ can be used as the score functions by
assuming that the probability density functions (pdfs) of the output signals $u$
approximate Laplacian and Gaussian distributions, respectively. Although the ICA-
based approach introduces the score function, additional computation is negligible
because output of the score function is commonly used for the learning rules of
all adaptive filter coefficients. Especially, computational requirements are reduced
with $\text{sign}(\cdot)$ as the score function because one multiplication can be replaced with
just sign change.

For the Laplacian source signals, the performances of the ICA-based approach
were better than those of the LMS algorithm. From these results, it can be rea-
soned that there may be many components in the primary input which depend on
the reference signal in higher-order statistics and these noise components can be
cancelled by the ICA-based learning rule. For the Gaussian source signals, how-
ever, the ICA-based approach provided almost the same SNRs as or a little worse
than the LMS algorithm. Gaussian signals can be described by only the first and
second-order statistics without higher-order statistics. Therefore, the ICA-based approach which can utilize higher-order statistics does not have any advantage over the LMS algorithm. If one use the score function which is not adequate to the original signal (for example, $\text{sign}(\cdot)$ for Gaussian signals), the performances can be degraded because the nonlinear function mismatches the cdf of the signal.

To perform experiments for real recorded signals, we used speech, car noise, or music as the noises. Another speech was used as the signal $s$. Korean sentences were recorded for the speech, and the car noise and the music were obtained in NOISEX-92 CD-ROMs and a Korean popular song, respectively. Each signal had 10 second length with $16kHz$ sampling rate. It is known that speech signal approximately follows Laplacian distribution. Therefore, $\text{sign}(\cdot)$ was used as the score function. Figure 1 shows the filter $h_{12}$ which was measured in a normal office room, and the number of taps of adaptive filter coefficients was 1024. Table 2 displays the SNRs of the two algorithms for the three different noises after convergence. The SNRs of the ICA-based approach were superior to those of the LMS algorithm. These results show that the ICA-based approach can remove dependent components through higher-order statistics for real recorded signals as well.

**Conclusion:** In this letter, a method for adaptive noise cancelling based on ICA was proposed and the ICA-based learning rule was derived. The method was compared with the LMS algorithm through the experiments for several noise signals and mixing filters. By including higher-order statistics, the proposed ICA-based
approach gave better performances than the conventional LMS algorithm.

Acknowledgments

This work was supported by the Brain Science & Engineering Research Program sponsored by Korean Ministry of Science and Technology.

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Table 1. SNRs of output signals for artificially generated i.i.d. signals after convergence (dB)

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<td>$\varphi = \text{sign}$</td>
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<tr>
<td>Laplacian</td>
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Table 2. SNRs of output signals for real recorded signals after convergence (dB)

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<tr>
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<tr>
<td>Speech</td>
<td>Music</td>
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Fig. 1. The measured filter in a normal office room