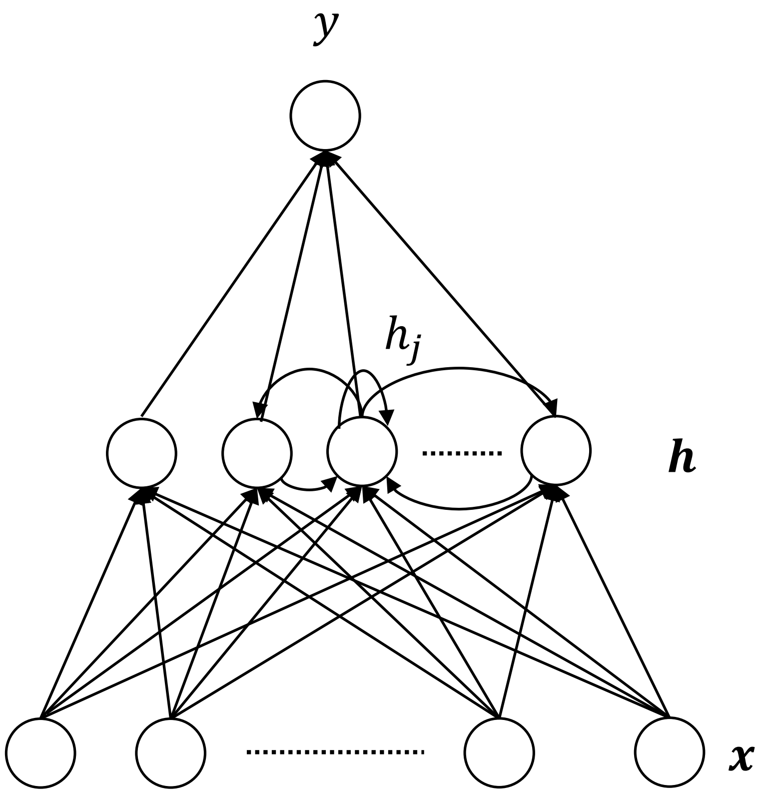
Machine Learning

Contents

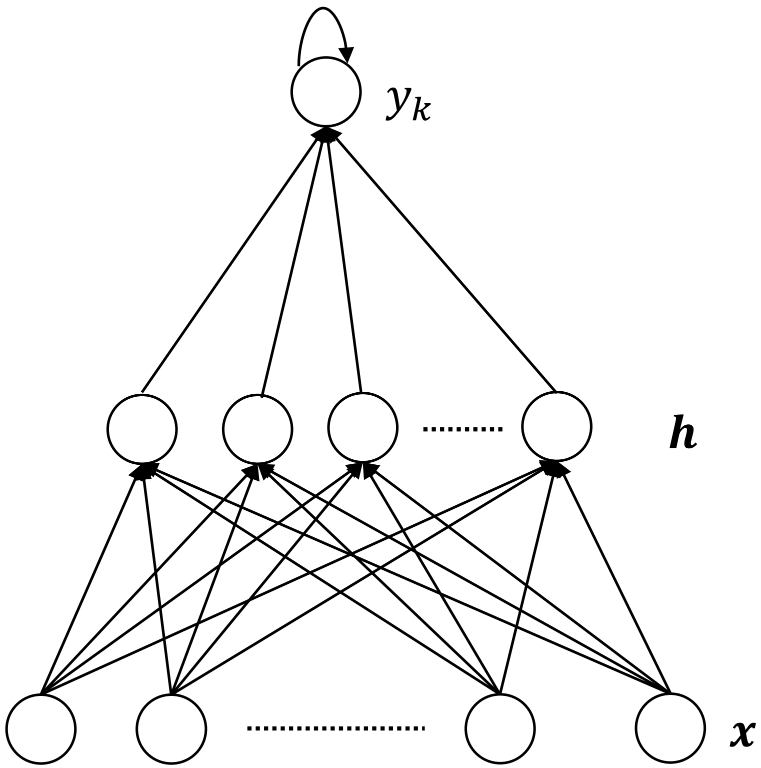
6.1. Recurrent Neural Networks

Universal approximation theorem for time series data



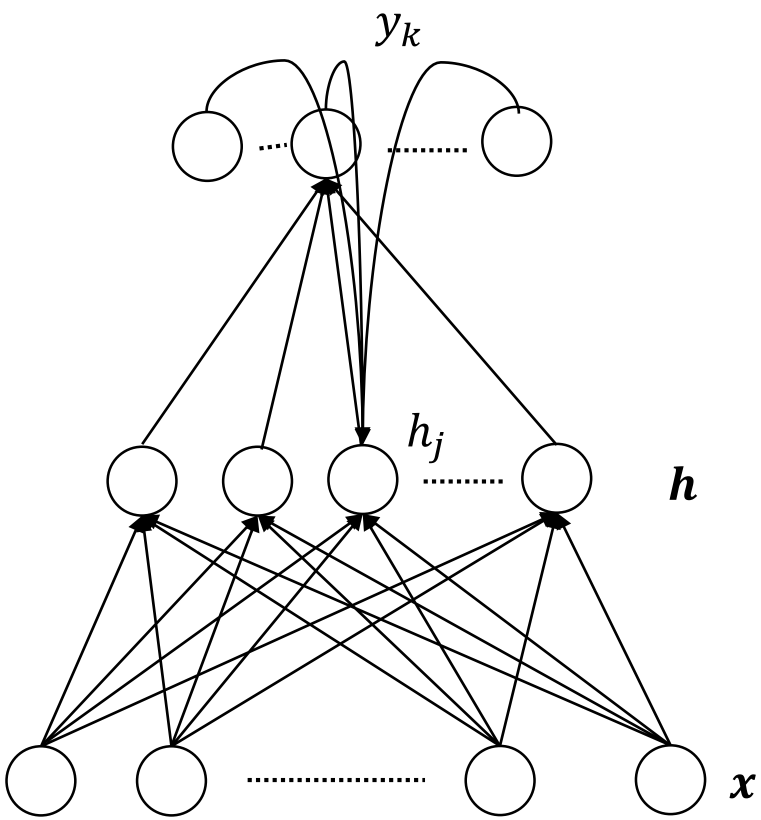
Recurrent connection for hidden nodes

$$\widehat{h_j}(t) = \sum_i w_{ji} x_i + \sum_k T_{jk} h_k(t-1) \quad (6.1.1)$$



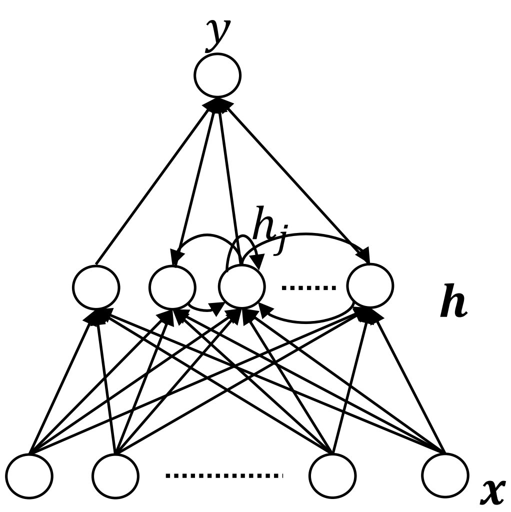
Recurrent connection for output nodes

$$\widehat{y_k}(t) = \sum_j v_{kj} h_j + T_k y_k(t-1) \quad (6.1.2)$$



Recurrent connection from output to hidden nodes

$$\hat{h}_j(t) = \sum_i w_{ji} x_i + \sum_k T_{jk} y_k (t-1) \tag{6.1.3}$$



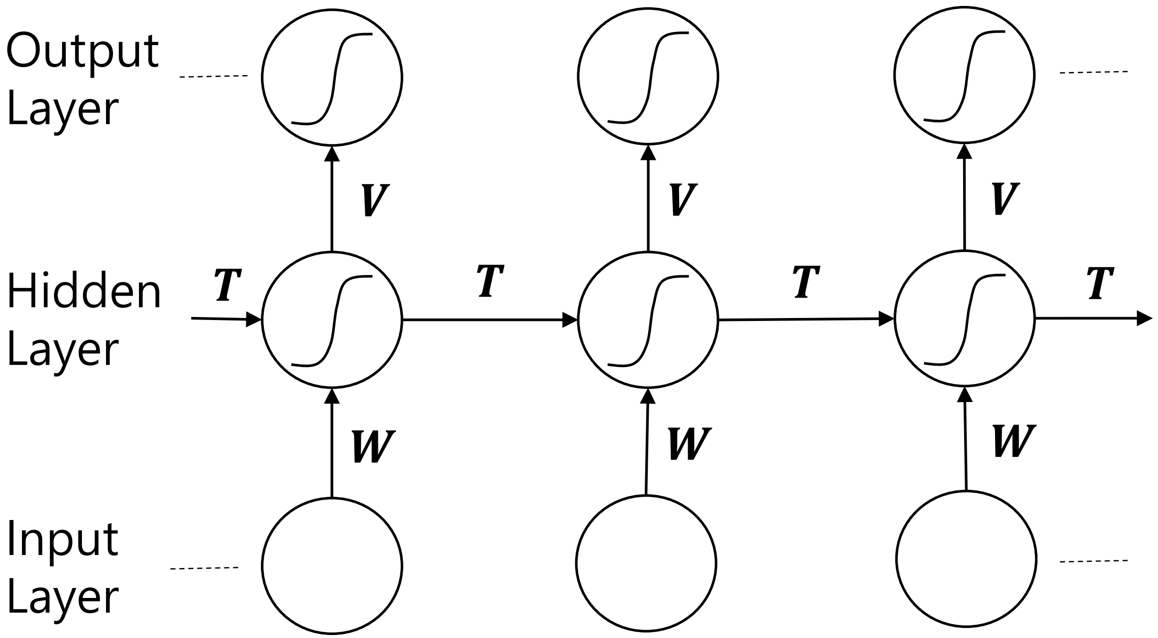


그림 6.4. 시간에 대하여 펼쳐진 은닉노드 회귀연결 신경회로망. 각 원은 특정 시간의 해당 층 노드들을 뜻함. **W**는 입력층에서 은닉층으로의 연결 가중치, **V**는 은닉층에서 출력층으로의 연결 가중치, **T**는 은닉층 내부의 회귀연결 가중치를 각각 뜻함.

$$\delta_j^{(hidden)}(t) = f'(.)\left(\sum_{k=1}^{M} v_{kj}\delta_k + \sum_{k=1}^{H} T_{jk}\delta_k^{(hidden)}(t+1)\right) \tag{6.1.4}$$

6.2. Long Short-Term Memory(LSTM)

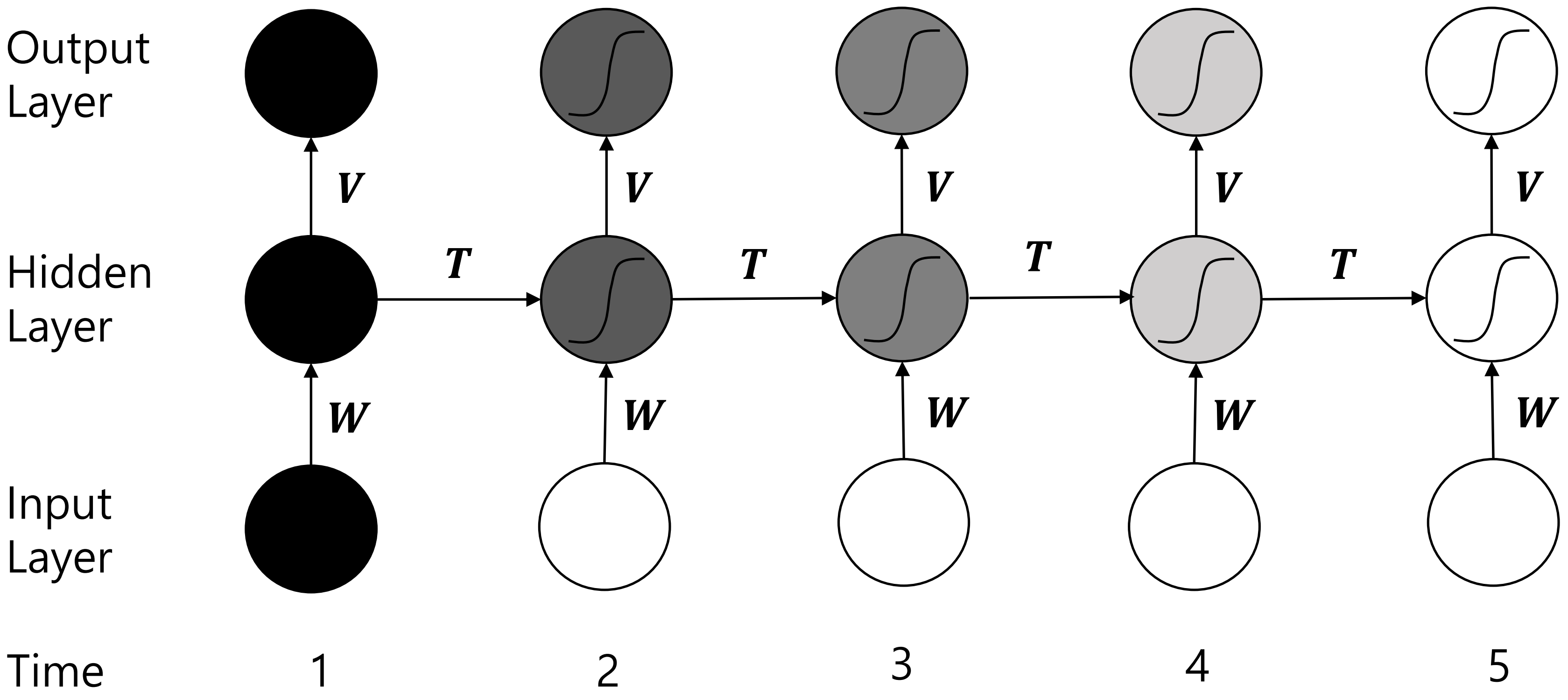


그림 6.5. 회귀신경회로망에서 시간에 따른 영향의 감소. 색깔의 강도는 민감도의 강도를 나타냄.

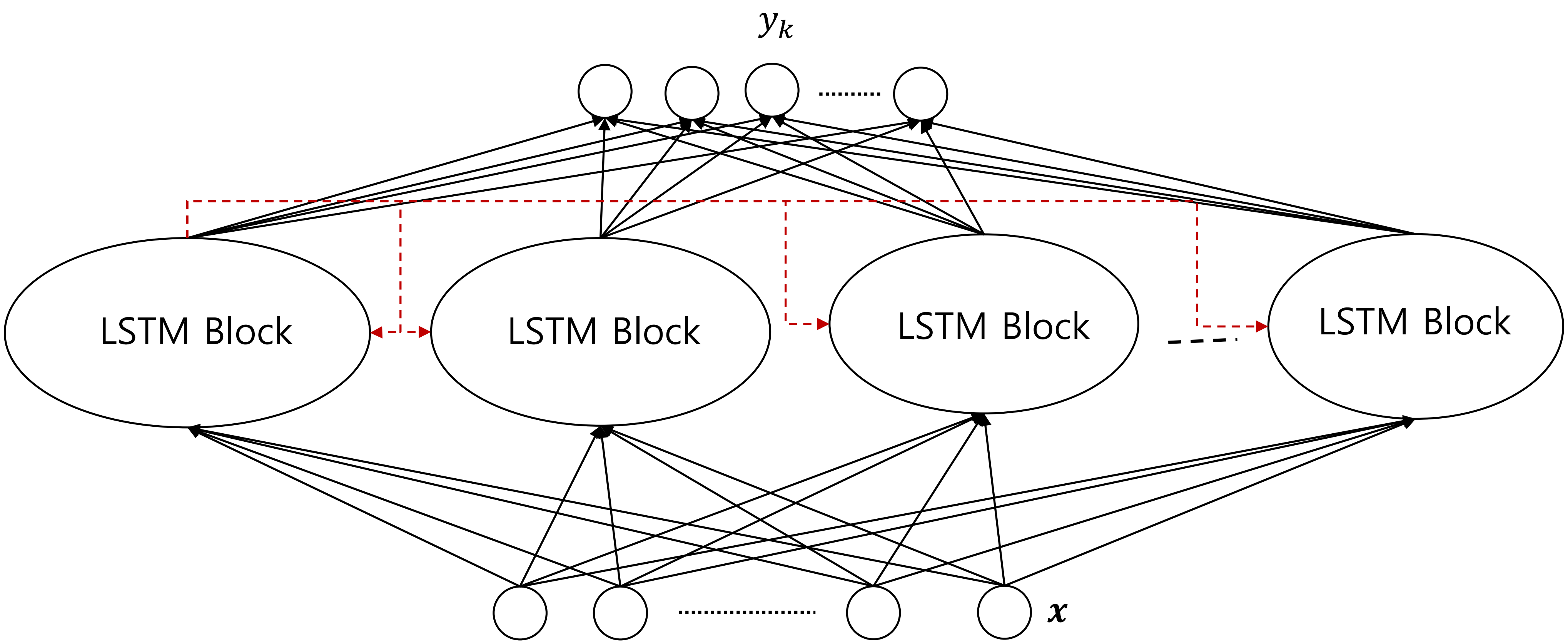
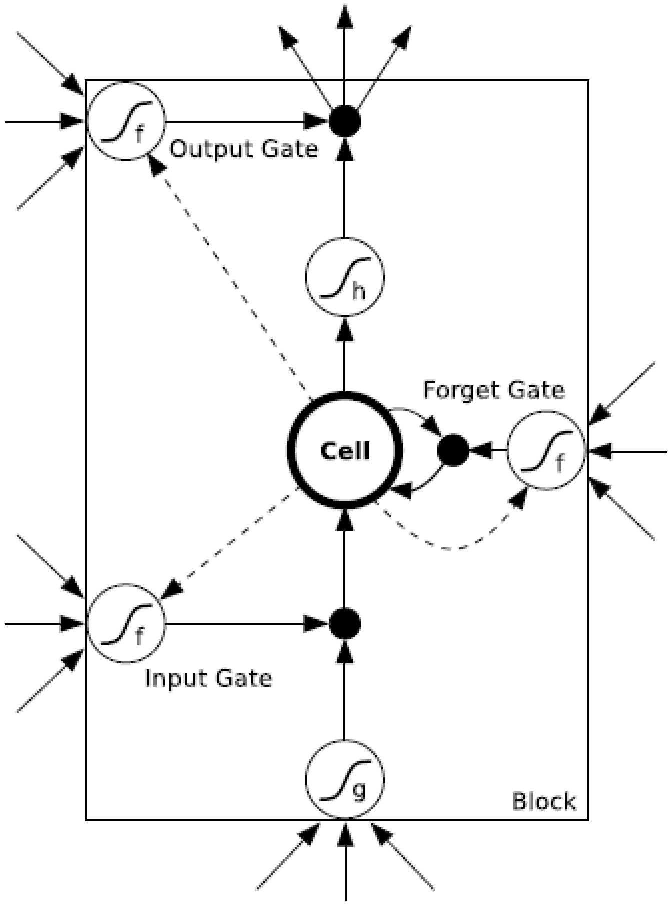


그림 6.6. LSTM 블록을 지닌 회귀 신경회로망. LSTM 블록 사이의 연결은 한 방향 만을 표현하였지만, 모든 LSTM 블록들은 서로 연결되어 있음.

Long Short-term Memory (LSTM, Hochreiterand Schmidhuber, 1997)

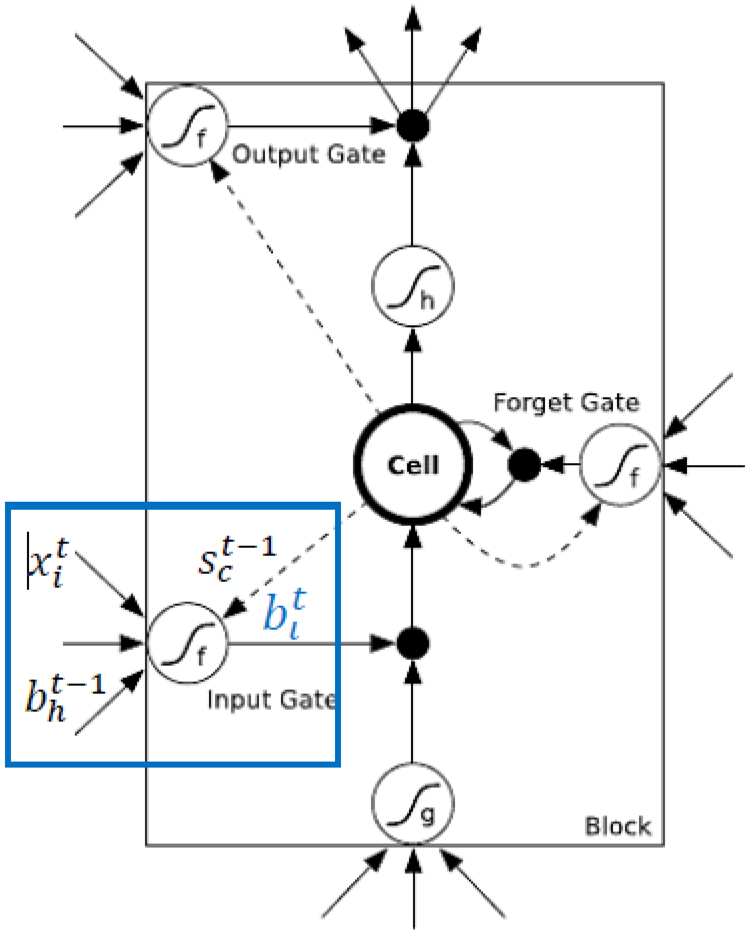
Consists of a set of recurrently connected subnets, known as **memory blocks**
Each block contains one or more self-connected memory **cells** and three multiplicative units –the **input, output, and forget gates.**
The three gates control the activation of the cell via multiplications



f: activation function of the gates
g: cell input activation function
h: cell output activation function

6.3. Forward Pass

Input Gates

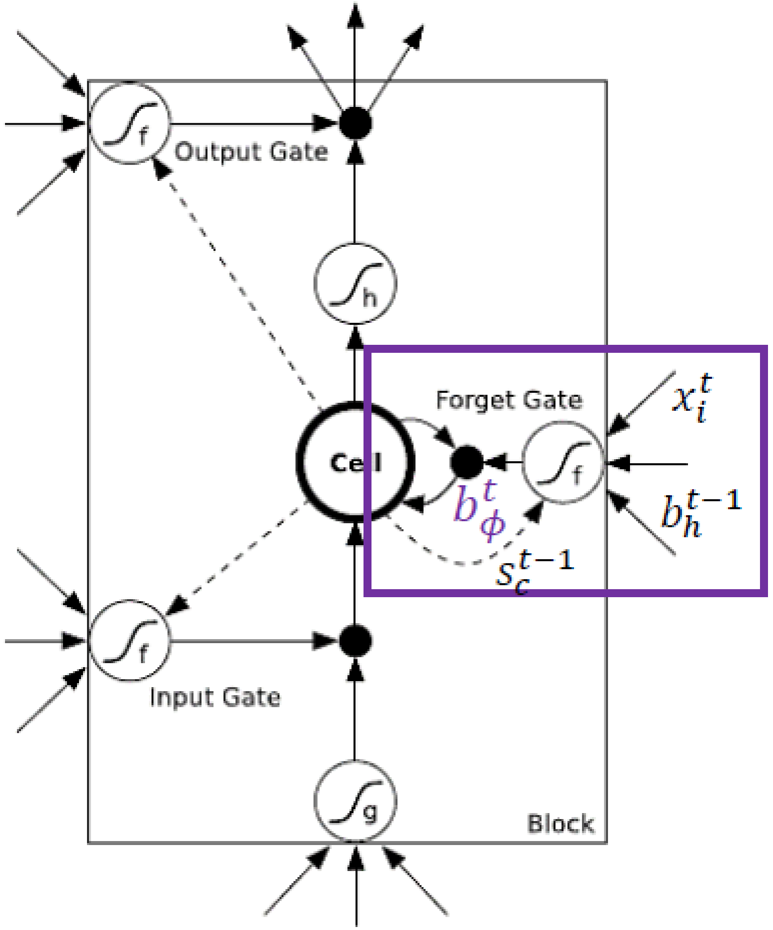


$$a_\iota^t = \sum_{i=1}^{I} w_{i\iota} x_i^t + \sum_{h=1}^{H} w_{h\iota} b_h^{t-1} + \sum_{c=1}^{C} w_{c\iota} s_c^{t-1} \tag{6.3.1}$$

$$b_\iota^t = f(a_\iota^t)$$

I : # inputs
K : # outputs
H : # cells in the hidden layer
C : # cells per a memory
G : total # inputs to the hidden layer (= 4H)

Forget Gates

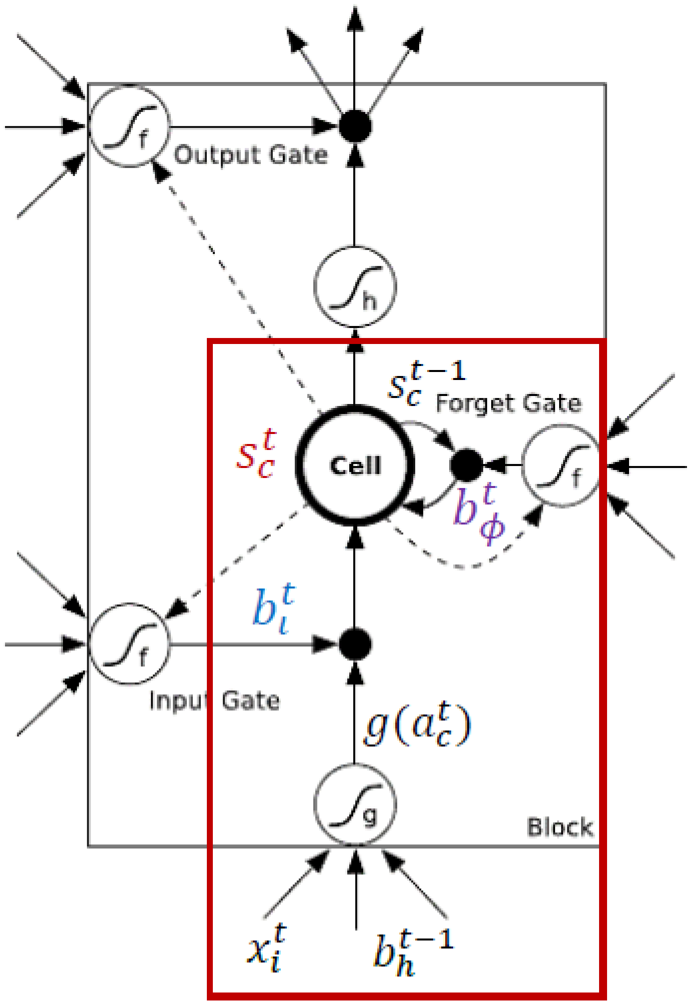


$$a_\phi^t = \sum_{i=1}^{I} w_{i\phi} x_i^t + \sum_{h=1}^{H} w_{h\phi} b_h^{t-1} + \sum_{c=1}^{C} w_{c\phi} s_c^{t-1} \quad (6.3.3)$$

$$b_\phi^t = f(a_\phi^t)$$

I : # inputs
K : # outputs
H : # cells in the hidden layer
C : # cells per a memory
G : total # inputs to the hidden layer (= 4H)

$Cells$

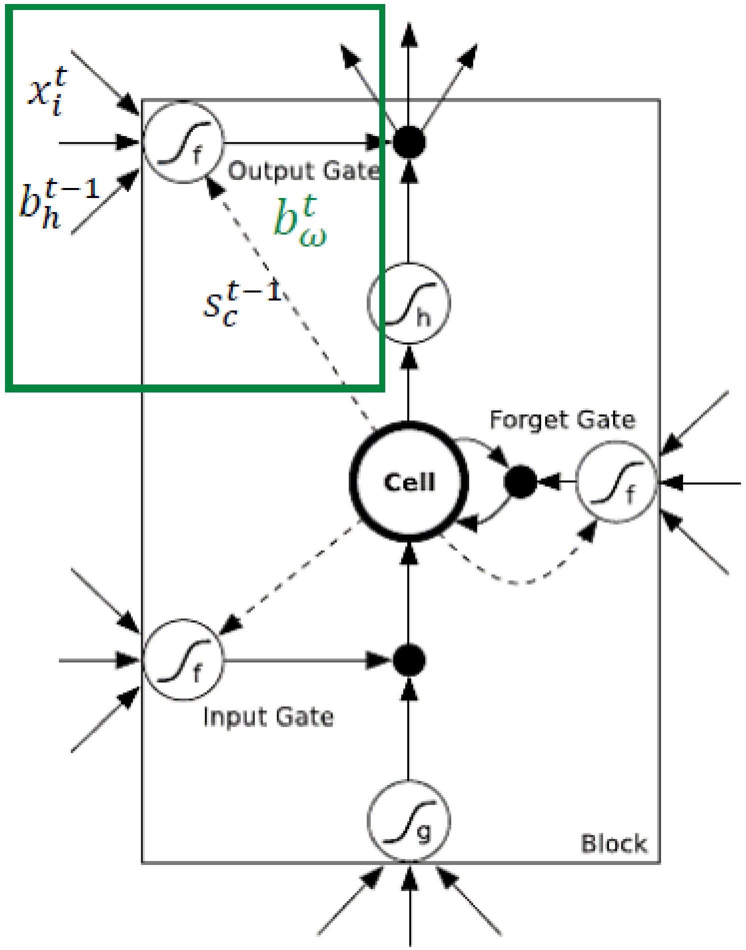


$$a_c^t = \sum_{i=1}^{I} w_{ic} x_i^t + \sum_{h=1}^{H} w_{hc} b_h^{t-1} \quad (6.3.5)$$

$$s_c^t = b_\phi^t s_c^{t-1} + b_\iota^t g(a_c^t) \quad (6.3.6)$$

I : # inputs
K : # outputs
H : # cells in the hidden layer
C : # cells per a memory
G : total # inputs to the hidden layer (= 4H)

$Output\ Gates$

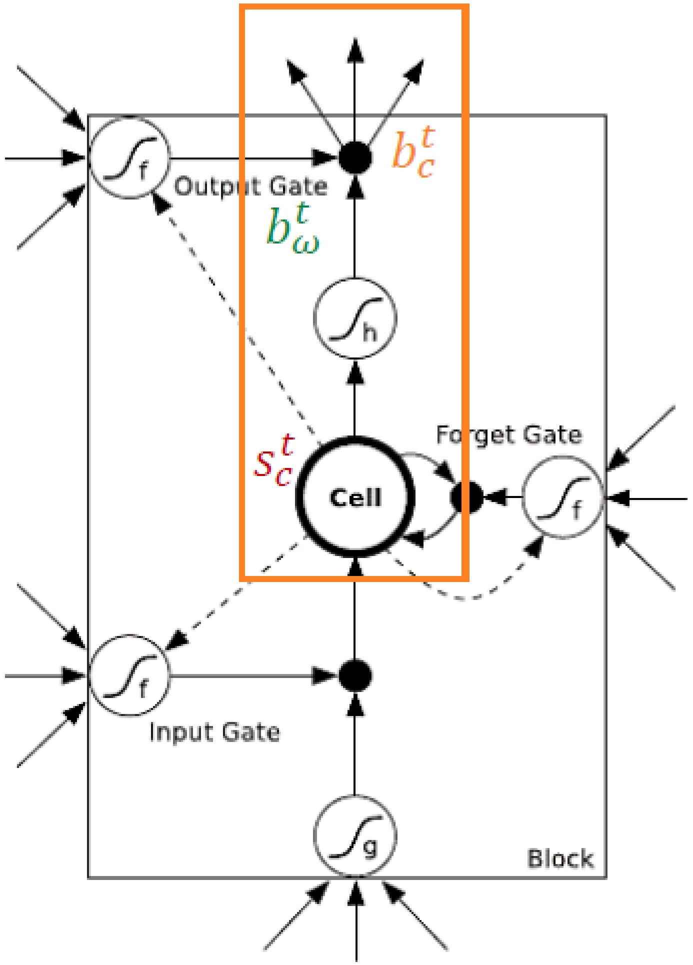


$$a_\omega^t = \sum_{i=1}^{I} w_{i\omega} x_i^t + \sum_{h=1}^{H} w_{h\omega} b_h^{t-1} + \sum_{c=1}^{C} w_{c\omega} s_c^t \quad (6.3.7)$$

$$b_\omega^t = f(a_\omega^t)$$

I : # inputs
K : # outputs
H : # cells in the hidden layer
C : # cells per a memory
G : total # inputs to the hidden layer (= 4H)

Cell Outputs



$$b_c^t = b_\omega^t h(s_c^t)$$

I : # inputs
K : # outputs
H : # cells in the hidden layer
C : # cells per a memory
G : total # inputs to the hidden layer (= 4H)

6.4. Backward Pass

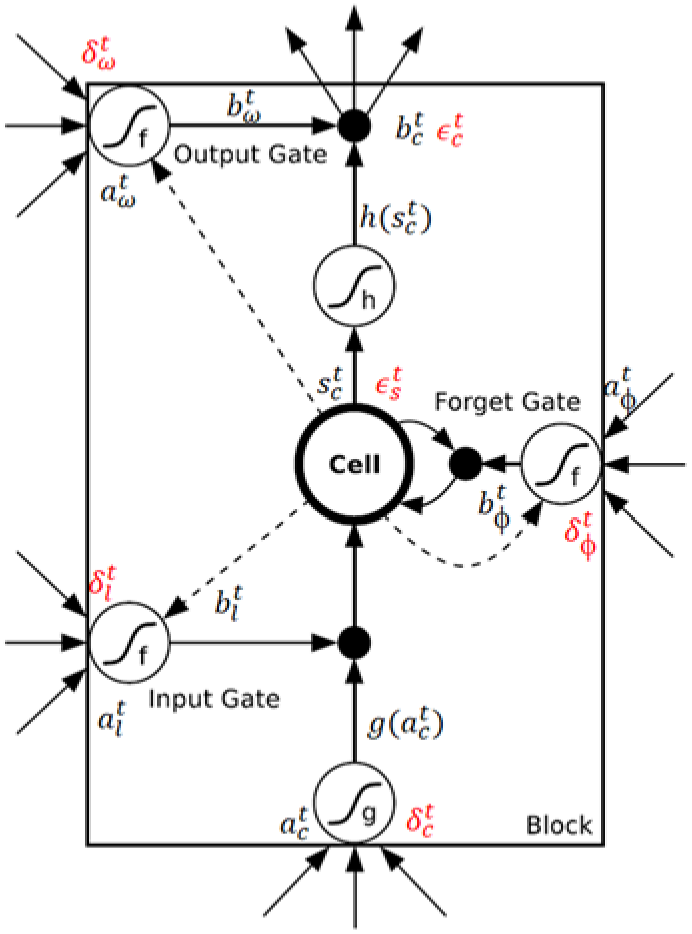


그림 6.12: 전방향 및 역방향 변수

Loss function: ***L***

Cell Outputs

$$\epsilon_c^t \equiv \frac{\partial L}{\partial b_c^t} = \sum_{k=1}^{K} w_{ck}\delta_k^t + \sum_{g=1}^{G} w_{cg}\delta_g^{t+1} \tag{6.4.1}$$

$$a_l^t = \sum_{i=1}^{I} w_{il}x_i^t + \sum_{h=1}^{H} w_{hl}b_h^{t-1} + \sum_{c=1}^{C} w_{cl}s_c^{t-1} \tag{6.3.1}$$

$$a_\phi^t = \sum_{i=1}^{I} w_{i\phi}x_i^t + \sum_{h=1}^{H} w_{h\phi}b_h^{t-1} + \sum_{c=1}^{C} w_{c\phi}s_c^{t-1} \tag{6.3.3}$$

$$a_c^t = \sum_{i=1}^{I} w_{ic}x_i^t + \sum_{h=1}^{H} w_{hc}b_h^{t-1} \tag{6.3.5}$$

$$a_w^t = \sum_{i=1}^{I} w_{iw}x_i^t + \sum_{h=1}^{H} w_{hw}b_h^{t-1} + \sum_{c=1}^{C} w_{cw}s_c^{t} \tag{6.3.7}$$

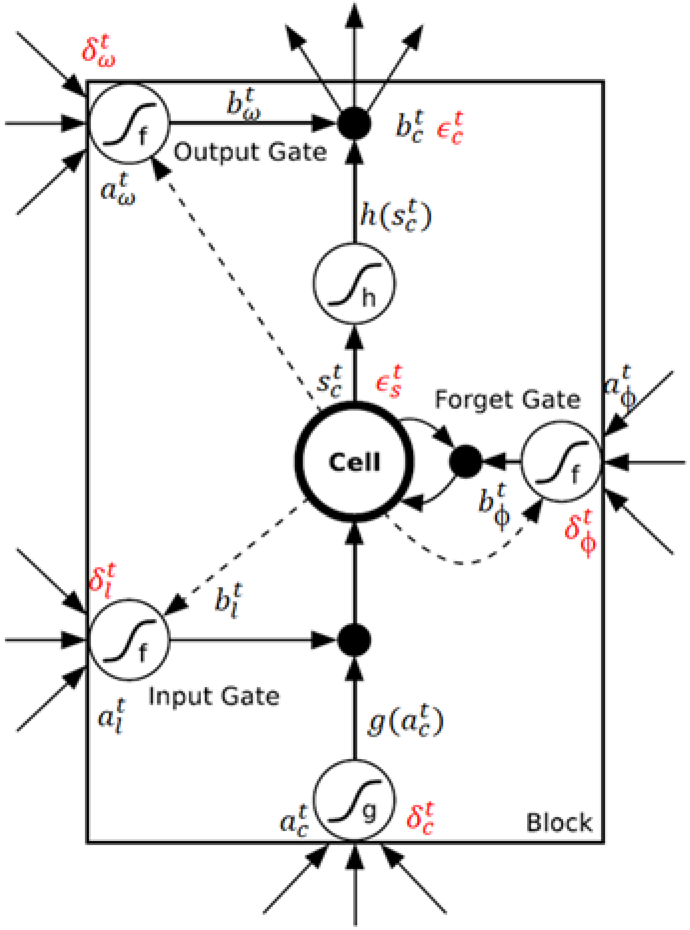


그림 6.12. 전방향 및 역방향 변수

Output Gates

$$\delta_w^t \equiv \frac{\partial L}{\partial a_w^t} = f'(a_w^t)\sum_{c=1}^{C} h(s_c^t)\epsilon_c^t \qquad (6.4.2)$$

States

$$\epsilon_s^t \equiv \frac{\partial L}{\partial s_c^t} = b_w^t h'(s_c^t)\epsilon_c^t + b_\phi^{t+1}\epsilon_s^{t+1} + w_{cl}\delta_l^{t+1} + w_{c\phi}\delta_\phi^{t+1} + w_{cw}\delta_w^t \qquad (6.4.3)$$

$$s_c^t = b_\phi^t s_c^{t-1} + b_l^t g(a_c^t) \qquad (6.3.6)$$

$$a_l^t = \sum_{i=1}^{I} w_{il}x_i^t + \sum_{h=1}^{H} w_{hl}b_h^{t-1} + \sum_{c=1}^{C} w_{cl}s_c^{t-1} \qquad (6.3.1)$$

$$a_\phi^t = \sum_{i=1}^{I} w_{i\phi}x_i^t + \sum_{h=1}^{H} w_{h\phi}b_h^{t-1} + \sum_{c=1}^{C} w_{c\phi}s_c^{t-1} \qquad (6.3.3)$$

$$a_w^t = \sum_{i=1}^{I} w_{iw}x_i^t + \sum_{h=1}^{H} w_{hw}b_h^{t-1} + \sum_{c=1}^{C} w_{cw}s_c^t \qquad (6.3.7)$$

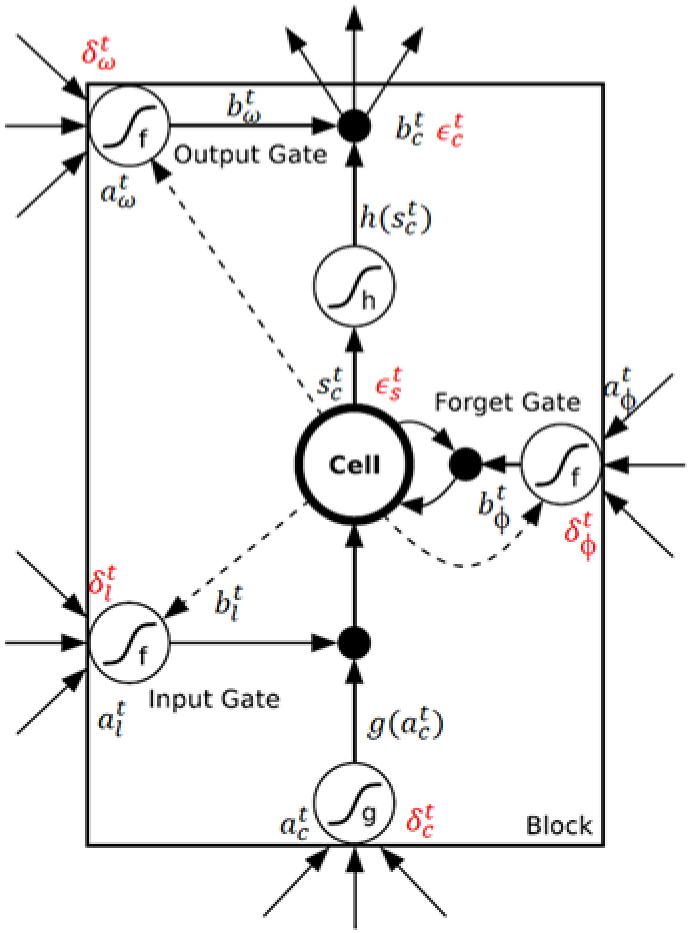


그림 6.12. 전방향 및 역방향 변수

Cells

$$\delta_c^t \equiv \frac{\partial L}{\partial a_c^t} = b_l^t g'(a_c^t)\epsilon_s^t \tag{6.4.4}$$

Forget Gates

$$\delta_\phi^t \equiv \frac{\partial L}{\partial a_\phi^t} = f'(a_\phi^t)\sum_{c=1}^{C} s_c^{t-1}\epsilon_s^t \tag{6.4.5}$$

Input Gates

$$\delta_l^t \equiv \frac{\partial L}{\partial a_l^t} = f'(a_l^t)\sum_{c=1}^{C} g(a_c^t)\epsilon_s^t \tag{6.4.6}$$

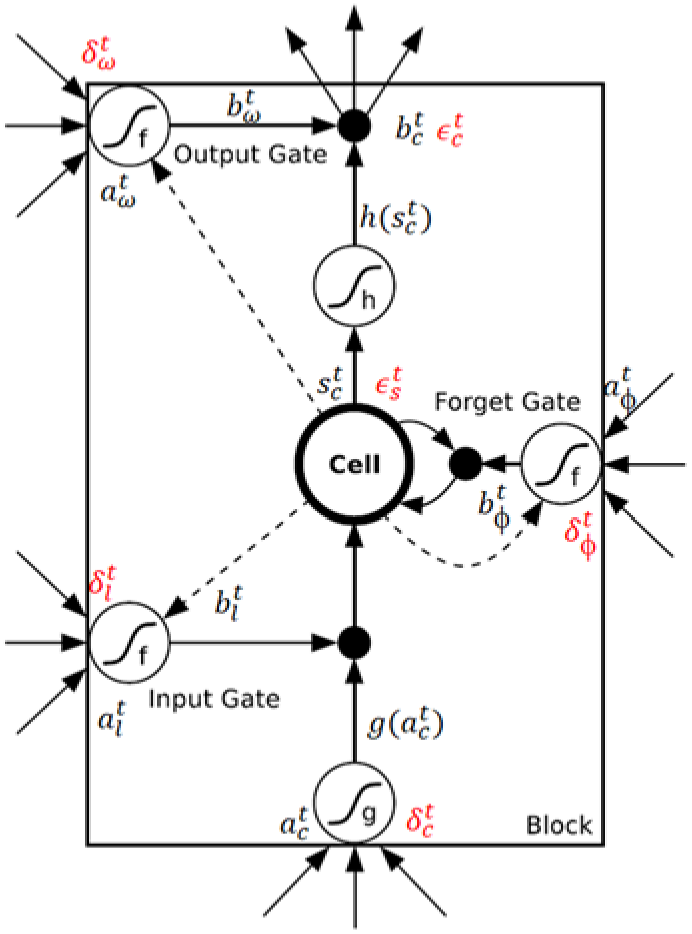


그림 6.12: 전방향 및 역방향 변수

$$\epsilon_c^t \stackrel{\text{def}}{=} \frac{\partial \mathcal{L}}{\partial b_c^t} \qquad \epsilon_s^t \stackrel{\text{def}}{=} \frac{\partial \mathcal{L}}{\partial s_c^t}$$

Cell Outputs

$$\epsilon_c^t = \sum_{k=1}^{K} w_{ck}\delta_k^t + \sum_{g=1}^{G} w_{cg}\delta_g^{t+1}$$

Output Gates

$$\delta_\omega^t = f'(a_\omega^t)\sum_{c=1}^{C} h(s_c^t)\epsilon_c^t$$

States

$$\epsilon_s^t = b_\omega^t h'(s_c^t)\epsilon_c^t + b_\phi^{t+1}\epsilon_s^{t+1} + w_{c\iota}\delta_\iota^{t+1} + w_{c\phi}\delta_\phi^{t+1} + w_{c\omega}\delta_\omega^t$$

Cells

$$\delta_c^t = b_\iota^t g'(a_c^t)\epsilon_s^t$$

Forget Gates

$$\delta_\phi^t = f'(a_\phi^t)\sum_{c=1}^{C} s_c^{t-1}\epsilon_s^t$$

Input Gates

$$\delta_\iota^t = f'(a_\iota^t)\sum_{c=1}^{C} g(a_c^t)\epsilon_s^t$$